Transmission lines: unravelling the mystery

By Harold Kinley

RF transmission lines have always presented an element of mystery. However, understanding the behavior of transmission lines is important to those of us who routinely work with these conduits of RF energy. Call them feedlines, transmission lines or simply "coax"—the *purpose* is to get energy from a transmitter output to an antenna, or from an antenna to a receiver input.

Non-resonant lines

When a transmission line is terminated in its characteristic impedance, all of the power delivered to the load by the incident (forward-traveling) wave is absorbed by the load. The line is said to be *non-resonant* in this situation. The impedance at each point along the line, at any point between the generator and the load, is the same. There is no reflected wave on the line. The impedance will be equal to the characteristic impedance of the line. The length of the line in this case is not important, except where loss is concerned. You might say that, compared to resonant lines, non-resonant lines are boring.

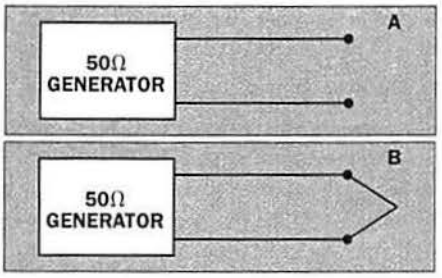

Figure 1. (A) A 50Ω transmission line that is unterminated, or open-circuited, at the end. The behavior of such a line is described in Table 1, below. (B) A 50Ω transmission line that is short-circuited at the end. The behavior of such a line is also described in Table 1.

Resonant lines

When a transmission line is terminated with an impedance that is *not* equal to the characteristic impedance of the line, things change. The impedance along the line between the generator and load varies but repeats at every half wavelength (λ/2) from the load. The load impedance may be purely resistive, or it may contain an element of reactance, either inductive or capacitive.

In Figure 1A at the left, a 50Ω transmission line (in a 50Ω radio system) is open-circuited at the load end. This is a theoretically perfect transmission line. It exhibits no loss and has a velocity factor of 1. Let's look down the line from the load end toward the generator, or source, in *one-thirty-second-wavelength* (l/32λ) increments. We only need to cover λ/2 of line because the situation starts repeating at λ/2. The second column in Table 1 below shows what the impedance will be at each incremental point on the line. Remember, this is for an open-circuited line—a near-perfect open circuit.

Now, if we place a short circuit on the end of the transmission line shown in Figure 1B, the situation changes. The third column of Table 1 (for the short-circuited line) lists the impedance value at each location moving from the end of the line toward the generator. This is for a near-perfect short circuit.

Some interesting points can be gleaned by studying Table 1. First, for the open-circuited line, the impedance consists of infinite resistance and infinite reactance at the end of the line. As the distance increases down the line (toward the generator), the resistive component becomes 0Ω and the inductive reactive component decreases until it reaches 0Ω at λ/4 from the end of the line. Thus, λ/4 from the end of the line, the impedance has shifted from an open circuit to a short circuit—a complete impedance transformation.

Now, moving further down the line from the end, between λ/4 and λ/2, the resistive component remains at 0Ω while the reactive component switches from inductive to capacitive and increases up to infinity (∞) at the λ/2 point. It is interesting to note that the reactive component is equal to the characteristic impedance of the line at both the λ/8 and 3λ/8 points. However, at the λ/8 point, the reactance is capacitive, but it is inductive at the 3λ/8 point. At the λ/2 point, the impedance again becomes infinity ($\infty \pm j\infty$). Thus, the impedance repeats at λ/2. This entire sequence would repeat for every λ/2 section of the transmission line. So, it is only necessary to study λ/2 section of

Table 1. The first column represents the distance from the end of the line in λ/32 wavelength increments. As shown here, the impedance repeats at every half wavelength and inverts at every quarter wavelength. These figures represent the situation with a lossless transmission line.

DISTANCE FROM LOAD (λ)	OPEN-CIRCUIT LINE IMPEDANCE (Ω)	SHORT-CIRCUIT LINE IMPEDANCE (Ω)
0/32	$\infty \pm j\infty$	$0 \pm j0$
1/32	$0 - j250$	$0 + j10$
2/32 = 1/16	$0 - j120$	$0 + j21$
3/32	$0 - j75$	$0 + j33$
4/32 = 1/8	$0 - j50$	$0 + j50$
5/32	$0 - j33$	$0 + j75$
6/32 = 3/16	$0 - j21$	$0 + j120$
7/32	$0 - j10$	$0 + j250$
8/32 = 1/4	$0 \pm j0$	$\infty \pm j\infty$
9/32	$0 + j10$	$0 - j250$
10/32 = 5/16	$0 + j21$	$0 - j120$
11/32	$0 + j33$	$0 - j75$
12/32 = 3/8	$0 + j50$	$0 - j50$
13/32	$0 + j75$	$0 - j33$
14/32 = 7/16	$0 + j120$	$0 - j21$
15/32	$0 + j250$	$0 - j10$
16/32 = 1/2	$\infty \pm j\infty$	$0 - j0$

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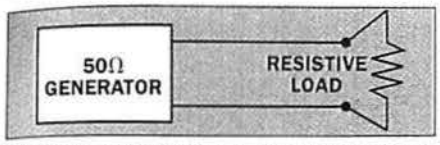

Figure 2. A 50Ω transmission line is terminated in a purely resistive load that is unequal to the characteristic impedance of the transmission line. See text for examples.

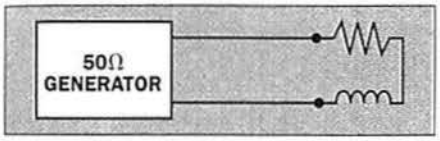

Figure 3: This 50Ω transmission line is terminated in a complex impedance load consisting of a 25Ω resistor and an inductive reactance of 25Ω. See text for details.

line to understand what is happening on the entire line.

The third column of Table 1 summarizes what happens with a short-circuited transmission line. Of course, at the short-circuited end, the impedance is $0 \pm j0\Omega$. Between the end of the line and the λ/4 point, the reactance is inductive. This is the reverse of the open-circuited line. Again, at the λ/8 and 3λ/8 points, the reactance is equal to the characteristic impedance of the line. At the λ/4 point, the impedance becomes an open circuit ($\infty \pm j\infty$). Between the λ/4 and λ/2 points, the reactance switches to *capacitive* reactance. And, at the λ/2 point, the short circuit at the end of the line is repeated and will do so at every multiple of λ/2 down the line toward the generator.

From this it can be concluded that for an open- or short-circuited transmission line, the impedance at any point between the end of the line and the λ/2 point will consist of 0Ω resistance and a reactive component that extends from 0 to infinity. Note that for an open-circuited line, the resistance component switches abruptly from 0Ω to infinity at the λ/2 point and every multiple thereof referenced to the end of the line. Conversely, for a short-circuited line, the resistance component switches abruptly from 0Ω to infinity at the λ/4 point and every odd multiple thereof referenced to the end of the line.

It can further be stated that if the load impedance is *purely resistive* and *greater than* the characteristic impedance of the line, the reactance will be *capacitive* at any point between the end of the line and λ/4 from the end, and *inductive* between the λ/4 and λ/2 points. If the load impedance is *purely resistive* and *less than* the characteristic impedance of the line, the reactance will be *inductive* at any point between the end of the line and the λ/4 point, and *capacitive* between the λ/4 and λ/2 points.

Figure 2 at the left illustrates a transmission line, with a characteristic impedance of 50Ω, terminated with a load that is a pure resistance of 25Ω ($25 \pm j0$). The VSWR will be 2:1. We can determine the impedance at λ/4 from the end of the line by using the following formula:

$$Z_i = \frac{Z^2}{Z_L}$$

where Z is the characteristic impedance of the line, Z_I is the impedance at the λ/4 point and Z_L is the load impedance. In Figure 2, the load impedance is a pure resistance of 25Ω and the characteristic impedance of the line is 50Ω. Now, plugging these values into the equation above we get:

$$Z_i = \frac{50^2}{25} = \frac{2,500}{25} = 100$$

Thus, the impedance at the λ/4 point is 100Ω. The VSWR will still be 2:1. *Assuming a lossless line, the VSWR at any point on a line will be the same.*

If in Figure 2 the 50Ω transmission line was terminated by a load with a pure resistance of 100Ω, the VSWR would still be 2:1. The impedance at a point λ/4 away from the end (according to the formula above) will be:

$$\frac{50^2}{100} = \frac{2,500}{100} = 25$$

Suppose the line is terminated in a complex impedance, one that contains both resistive and reactive components. The formula still holds true, but now we must enter the load impedance in complex form. In Figure 3, a 50Ω transmission line is terminated in an impedance of $25 + j25$ ohms. Plugging this into the formula we get:

$$Z = \frac{50^2}{(25+j25)} = \frac{2,500}{(25+j25)} = \frac{2,500}{25(1+j)} = \frac{100}{(1+j)}$$

$$= \frac{100}{(1+j)} \times \frac{(1-j)}{(1-j)} = \frac{100(1-j)}{2} = 50 - j50$$

Thus, the impedance λ/4 down the line is equal to $50 - j50\Omega$. The inductive reactance present in the load impedance has been transformed into capacitive reactance at the λ/4 point down the line. To reiterate, *although the impedance on the line changes, the VSWR does not change.*

Next month's column deals with practical applications of this information.

So, until next time ... *stay tuned!* ■

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Working with RF transmission lines

By Harold Kinley

As stated in January's "Technically Speaking," when a transmission line is terminated in its characteristic impedance, the line is *non-resonant*. The length of a non-resonant line is not important, except when considering the attenuation of the desired signal between the transmitter and the antenna, or between the antenna and the receiver. The impedance anywhere along the line will be the same.

Standard system impedance

In the land mobile radio field the standard impedance is 50Ω. Mobile radio transmitters and power amplifiers are designed to work into a 50Ω load impedance. Antennas are designed to present a 50Ω impedance to the transmission line. The coaxial cable that delivers power from the transmitter or amplifier to the antenna has a characteristic impedance of 50Ω.

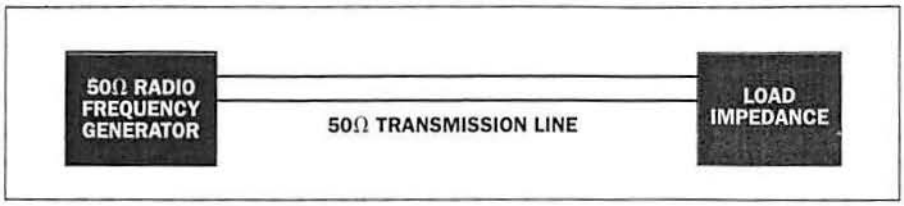

Figure 1. A 50Ω signal generator is connected to a load impedance through a 50Ω transmission line. The load impedance can be anything from a short circuit to an open circuit and everything between those two extremes.

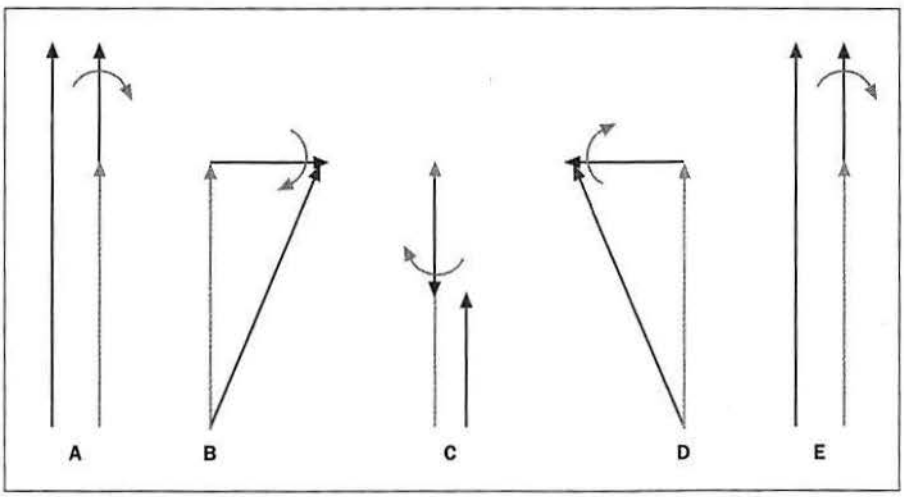

Figure 2. These phasors indicate how the standing wave is constructed. The green phasor represents the voltage of the incident wave, while the red phasor represents the voltage of the reflected wave. The vector *resultant* (blue) represents the voltage of the standing wave. Because the green phasor is the reference, it is shown stationary, with the red phasor revolving around it. This situation represents a 50Ω transmission line that is connected to a purely resistive load of 150Ω. This represents a VSWR of 3:1.

(A) The end of the line. Here, the reflected wave and the incident wave are in phase and the voltages of the two waves add to produce the resultant vector, shown in blue. Thus, the voltage of the standing wave is equal to the sum of the voltages of both the incident and the forward waves. Because the voltage of the reflected wave is half the voltage of the incident wave, the resultant at (A) will be ×1.5 the voltage of the reflected wave.

(B) 45°, or λ/8, down the line from the load, the two phasors are 90° out-of-phase, resulting in a standing wave voltage that is ×1.12 the voltage of the incident wave.

(C) 90°, or λ/4, down the line from the load, the two phasors are 180° out-of-phase, resulting in a standing wave voltage that is half the voltage of the incident wave.

(D) 135°, or 3λ/8, down the line from the load, the two phasors are again 90° out-of-phase, resulting in a standing wave voltage that is equal to ×1.12 the voltage of the incident wave. (E) 180°, or λ/2, down the line from the load, the two phasors are again in phase, resulting in a standing wave voltage that is equal to ×1.5 the voltage of the incident wave.

This series of phasors demonstrates the construction of the standing wave voltage from maximum through minimum and back to maximum. The cycle is repeated every λ/2 down the line.

In Figure 1 below, a 50Ω generator is connected to a 50Ω transmission line that is terminated in a load. If the impedance of the load is 50Ω (purely resistive), all of the power delivered to the load will be dissipated in the load. If the load impedance is anything other than 50Ω (purely resistive), power will be reflected back down the transmission line toward the source or generator.

If the impedance mismatch is a worst-case mismatch (open or short circuit), then all of the power delivered to the load will be reflected back down the line toward the source or generator. Thus, two traveling waves will be on the transmission line: a *forward*-traveling wave and a *reflected*-traveling wave. The forward traveling wave is often referred to as the *incident* wave. The phase relationship of the incident and reflected waves will vary along the length of the line.

Load mismatch

For this discussion, assume a lossless transmission line terminated with a purely resistive load of 150Ω. Let's consider the voltage levels of the two waves. For this discussion, let's assign 1.0V as the voltage of the incident wave. The voltage of the incident wave at the load will be 1.0V, but what about the voltage of the reflected wave at that point? We know that the VSWR will be 150 ÷ 50 = 3:1. By using a unit called the *reflection coefficient*, we can determine the voltage of the reflected wave. Reflection coefficient can be determined from the VSWR by the formula:

$$\Gamma = \frac{S-1}{S+1}$$

where Γ = *reflection coefficient*
S = *VSWR.*

From this we can determine that the reflection coefficient is 0.5. If we multiply the voltage of the incident wave (1.0V) by the reflection coefficient, we find that the voltage of the reflected wave is 0.5V. When the load impedance

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Table 1. The reflection coefficient, phase angle and resultant voltage of the standing wave for a purely resistive load of 150Ω in a 50Ω system.

Distance from load in wavelengths (λ)	Reflection coefficient	Phase angle	Voltage of standing wave
0λ	0.5	0°	1.5V
0.025λ	0.5	−18°	1.48V
0.05λ	0.5	−36°	1.43V
0.075λ	0.5	−54°	1.36V
0.10λ	0.5	−72°	1.25V
0.125λ	0.5	−90°	1.12V
0.150λ	0.5	−108°	0.97V
0.175λ	0.5	−126°	0.81V
0.20λ	0.5	−144°	0.66V
0.225λ	0.5	−162°	0.55V
0.25λ	0.5	−180°	0.50V
0.275λ	0.5	−198°	0.55V
0.30λ	0.5	−216°	0.66V
0.325λ	0.5	−234°	0.81V
0.35λ	0.5	−252°	0.97V
0.375λ	0.5	−270°	1.12V
0.40λ	0.5	−288°	1.25V
0.425λ	0.5	−306°	1.36V
0.45λ	0.5	−324°	1.43V
0.475λ	0.5	−342°	1.48V
0.50λ	0.5	−360°	1.50V

Table 2. The reflection coefficient, phase angle and resultant voltage of the standing wave for a purely resistive load of 16.67Ω in a 50Ω system.

Distance from load in wavelengths (λ)	Reflection coefficient	Phase angle	Voltage of standing wave
0λ	0.5	180°	0.5V
0.025λ	0.5	162°	0.55V
0.05λ	0.5	144°	0.66V
0.075λ	0.5	126°	0.81V
0.10λ	0.5	108°	0.97V
0.125λ	0.5	90°	1.12V
0.150λ	0.5	72°	1.25V
0.175λ	0.5	54°	1.36V
0.20λ	0.5	36°	1.43V
0.225λ	0.5	18°	1.48V
0.25λ	0.5	0°	1.50V
0.275λ	0.5	−18°	1.48V
0.30λ	0.5	−36°	1.43V
0.325λ	0.5	−54°	1.36V
0.35λ	0.5	−72°	1.25V
0.375λ	0.5	−90°	1.12V
0.40λ	0.5	−108°	0.97V
0.425λ	0.5	−126°	0.81V
0.45λ	0.5	−144°	0.66V
0.475λ	0.5	−162°	0.55V
0.50λ	0.5	−180°	0.50V

is greater than the system impedance, the voltage of the reflected wave is in phase with the voltage of the incident wave at the load. It is the combination (vector sum) of the incident voltage and the reflected voltage that produces the standing wave voltage. Refer to Figure 2 on page 20. Because, in this case, the reflected and forward voltages are in phase at the load, the voltage of the standing wave at this point will be 1V + 0.5V = 1.5V.

Reflection coefficient also has an angle associated with it. This is the phase angle of the reflected wave as referenced to the incident wave. When the reflected wave is in phase, or 180° out of phase with the incident wave, the resultant standing wave voltage is determined simply by either adding the two (for in phase) or subtracting the reflected voltage from the incident voltage (for 180° out of phase).

When the phase angle is other than 0° or 180°, we must add the two, using vectors to find the resultant voltage in

the standing wave.

Reflection coefficient can also be determined easily from reflected- and forward-power measurements. Divide the reflected power by the forward power and find the square root of the result. For example, 100W forward power and 25W reflected power yields a ratio of 0.25. The square root of 0.25 is 0.5—the reflection coefficient.

Table 1 on page 22 shows the reflection coefficient, phase angle and resultant voltage of the standing wave for a purely resistive load of 150Ω in a 50Ω system. Figure 3 at the right graphs the standing wave using data in Table 1.

In the previous example, the load impedance was purely resistive and, at 150Ω, three times greater than the system impedance. Let's examine a situation where the load impedance is less than (1/3) the system impedance, or 16.67Ω. Again, the VSWR will be 3:1, and the reflection coefficient will be 0.5. Table 2 on page 22 shows the data for reflection angle, voltage of standing wave (assuming a voltage level of 1V for the incident wave) and reflection coefficient at various distances

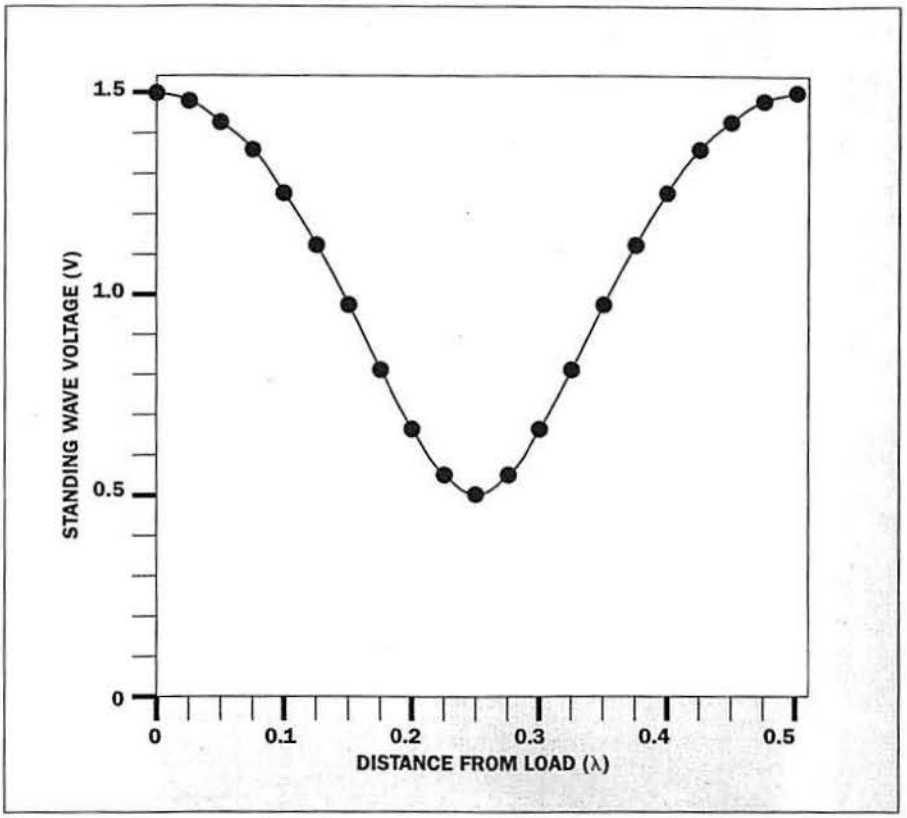
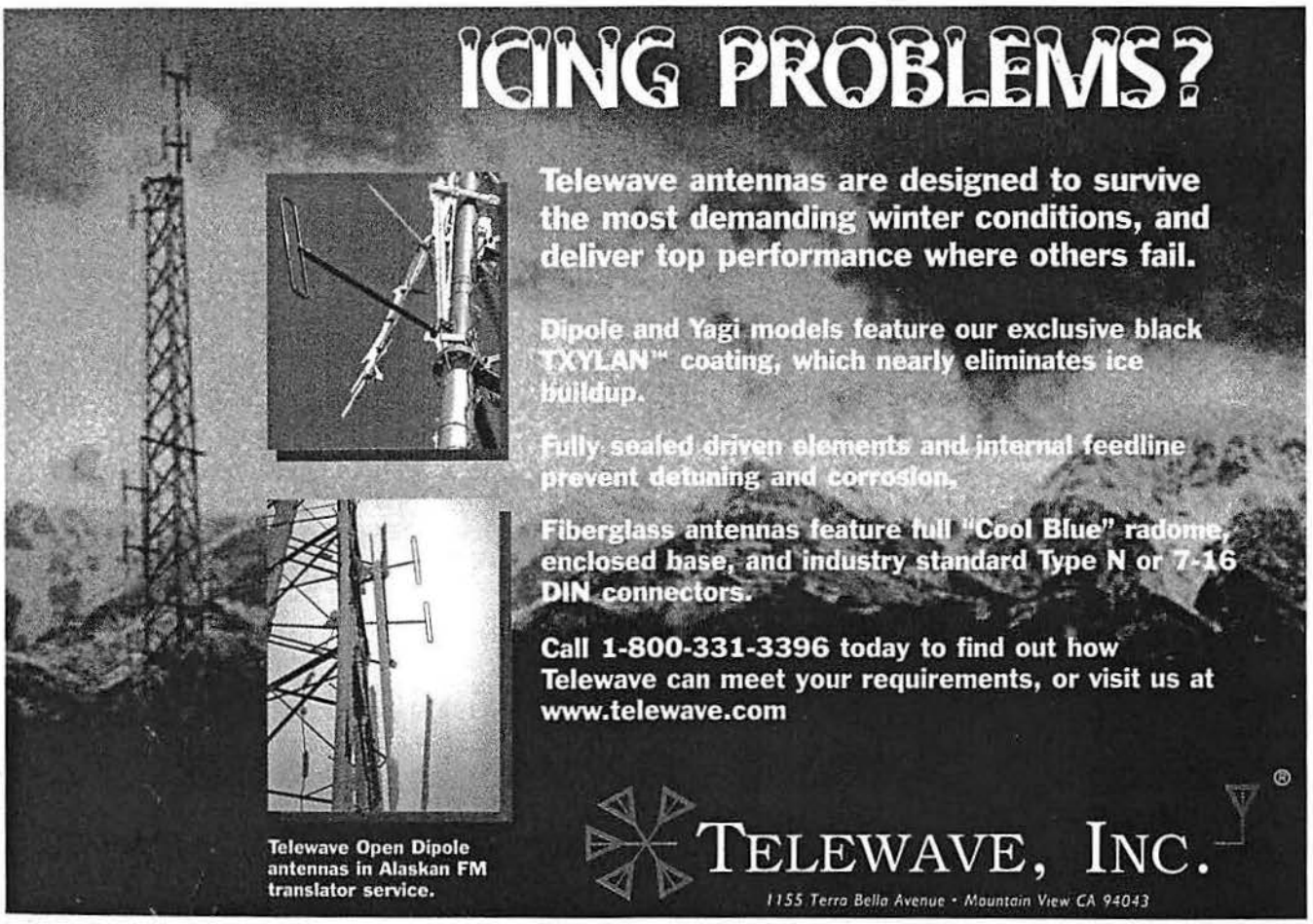


Figure 3. The impedance mismatch caused by the 150Ω load creates the VSWR pattern shown here. At the load, the voltage in the standing wave is at a maximum point (maxima). At λ/4 down the line, the voltage in the standing wave is at a minimum (minima). At λ/2 down the line, the voltage is again at a maximum. Because the VSWR is equal to the voltage maxima divided by the voltage minima, the VSWR is 1.5 ÷ 0.5 = 3:1.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from the load. Figure 4 at the right graphs the data from Table 2.

Compare Figures 3 and 4. When the load impedance is lower than the characteristic impedance of the transmission line, the voltages of the reflected and incident waves are 180° out of phase at the load. This results in a voltage *minima* on the standing wave pattern. The voltage maxima will occur at a point a quarter wavelength (λ/4) down the line and every λ/2 from that point. This is the reverse of the condition where the load resistance is higher than the characteristic impedance of the line. In that case, the voltage of the standing wave is at a maxima at the load because the reflected and incident waves are in phase, producing a higher resultant voltage on the standing wave pattern.

The Smith chart has long been recognized as one of the most valuable tools invented for transmission line calculations. Next month we will examine a software version of the famous Smith chart. You might be surprised to learn that the Smith chart is not for RF design engineers only.

So, until next time—*stay tuned!* ■

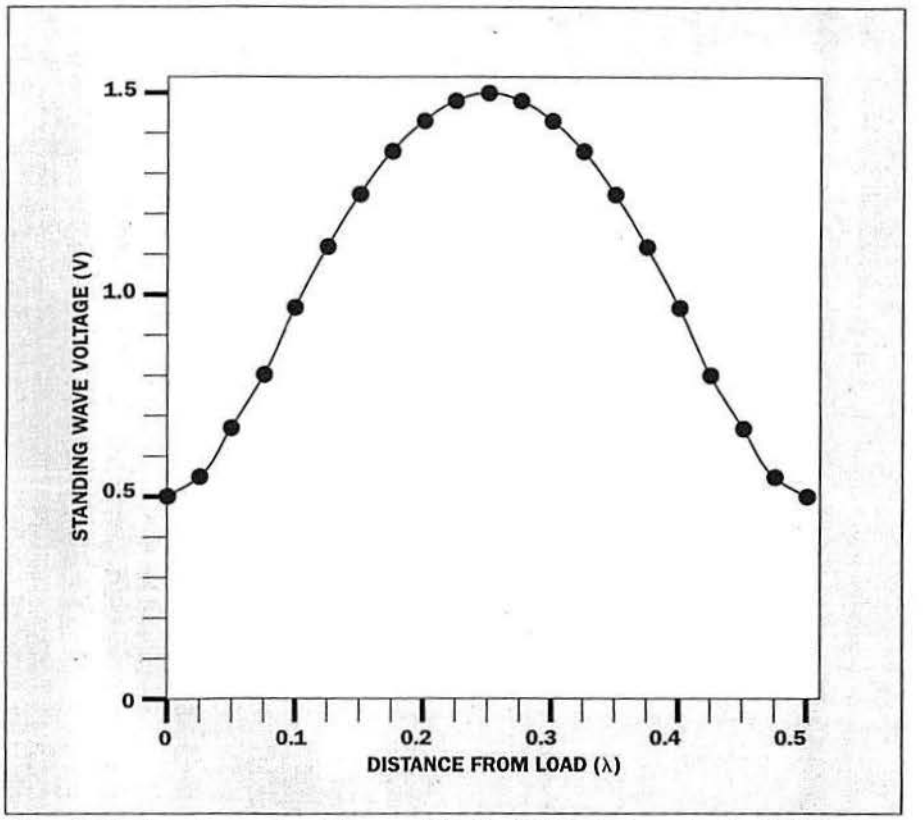
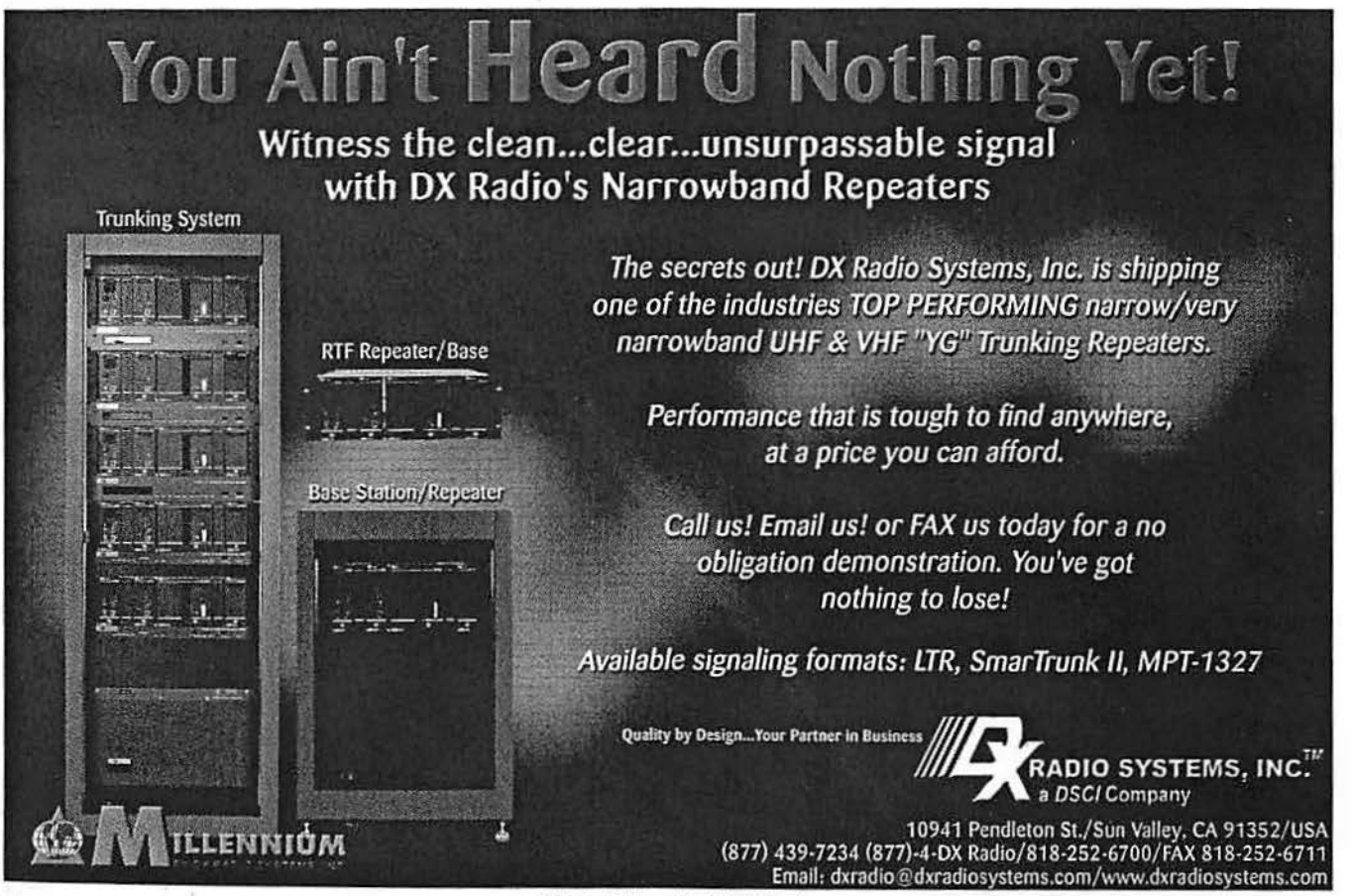


Figure 4. The impedance mismatch caused by the 16.67Ω load creates the VSWR pattern shown here. At the load, the voltage in the standing wave is at a minima. At λ/4 down the line, the voltage is at a maxima. At λ/2 down the line, the voltage is at a minima again. Again, the VSWR is equal to the maxima divided by the minima, or 1.5 ÷ 0.5 = 3:1.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